B.Sc. B.Ed SEM-I Examination: 2020

Course-CC1

Subject: Mathematics (Elementary Algebra and Calculus)

Time: 2 Hours   F.M. 50

Answer any *ten* questions $(5 \times 10 = 50)$

1. State De-Moivre's theorem for positive integer indices. Using this theorem, find the value of $\frac{3}{(2+i)^3}$ 2 + 3

2. If α, β, γ be the roots of the equation $x^3 + x + 1 = 0$, then prove that $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1) = 1$.   5

3. If a, b, c be positive and $a + b + c = 1$, then show that, $(\frac{1}{a} - 1)(\frac{1}{b} - 1)(\frac{1}{c} - 1) \geq 8$.   5

4. Solve the equation: $U_{n+2} - 7U_{n+1} - 8U_n = n^2 . 2^n$

5. Define Hermitian and Skew-Hermitian matrices. Show that any square matrix A can be written as the sum of a Hermitian and Skew-Hermitian matrices.   2 + 3

6. If possible, solve the system of equations:

   $2x + y - 3z = 8$

   $x - y - 2z = -2$

   $x + 2y - z = 10$ 5

7. State $\mathcal{E} - \delta$ definition of limit of a function. Then show that $\lim_{x \to 0} f(x) = 4$ where, $f(x) = \frac{x^2 - 4}{x - 2}, x \neq 2$ 2 + 3

8. Examine the nature of discontinuity of $f(x) = [x] + [1 - x]$ at $x = 0$ 5

9. A function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = |x| + |x - 1| + |x - 2|$. $x \in \mathbb{R}$. Find $f'(x)$ and specify the domain.   5

10. If $y = \frac{1}{x^2 + a^2}$, prove that $y_n = \frac{(-1)^n n!}{a^{n+2}} sin^{n+1}\theta sin(n + 1)\theta$ where $cot\theta = \frac{x}{a}$ 5

11. Find the value of $\lim_{x \to 0} \frac{tanx - x}{x^2 tanx}$ 5

12. If $I_{m,n} = \int_0^{\frac{\pi}{2}} sin^m x cos\, nx\, dx$ then show that

    $(m + n)I_{m,n} = sin\frac{n\pi}{2} - mj_{m-1,n-1}$

    Where, $j_{m,n} = \int_0^{\frac{\pi}{2}} sin^m x sin\, nx\, dx$ $(m \geq 1)$ 5